# Artificial intelligence 1: informed search

Lecturer: Tom Lenaerts
SWITCH, Vlaams Interuniversitair Instituut voor Biotechnologie

Vrije Universiteit Brussel

## Outline

- Informed = use problem-specific knowledge
- Which search strategies?
  - Best-first search and its variants
- Heuristic functions?
  - How to invent them
- Local search and optimization
  - Hill climbing, local beam search, genetic algorithms,…
- Local search in continuous spaces
- Online search agents



## Previously: tree-search

**function** TREE-SEARCH(*problem,fringe*) **return** a solution or failure
    *fringe* ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
    **loop do**
        **if** EMPTY?(*fringe*) **then return** failure
        *node* ← REMOVE-FIRST(*fringe*)
        **if** GOAL-TEST[*problem*] applied to STATE[*node*] succeeds
            **then return** SOLUTION(*node*)
        *fringe* ← INSERT-ALL(EXPAND(*node*, *problem*), *fringe*)

A strategy is defined by picking *the order of node expansion*



## Best-first search

- General approach of informed search:
  - Best-first search: node is selected for expansion based on an *evaluation function f(n)*
- Idea: evaluation function measures distance to the goal.
  - Choose node which *appears* best
- Implementation:
  - *fringe* is queue sorted in decreasing order of desirability.
  - Special cases: greedy search, A* search



## A heuristic function

[dictionary]*"A rule of thumb, simplification, or educated guess that reduces or limits the search for solutions in domains that are difficult and poorly understood."*

- $h(n)$ = estimated cost of the cheapest path from node $n$ to goal node.
- If $n$ is goal then $h(n)=0$

More information later.



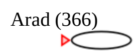
## Romania with step costs in km

| | | | |
|---|---|---|---|
| **Arad** | 366 | **Mehadia** | 241 |
| **Bucharest** | 0 | **Neamt** | 234 |
| **Craiova** | 160 | **Oradea** | 380 |
| **Dobreta** | 242 | **Pitesti** | 100 |
| **Eforie** | 161 | **Rimnicu Vilcea** | 193 |
| **Fagaras** | 176 | **Sibiu** | 253 |
| **Giurgiu** | 77 | **Timisoara** | 329 |
| **Hirsova** | 151 | **Urziceni** | 80 |
| **Iasi** | 226 | **Vaslui** | 199 |
| **Lugoj** | 244 | **Zerind** | 374 |

- $h_{SLD}$=straight-line distance heuristic.
- $h_{SLD}$ can **NOT** be computed from the problem description itself
- In this example $f(n)=h(n)$
  - Expand node that is closest to goal

  = Greedy best-first search

## Greedy search example

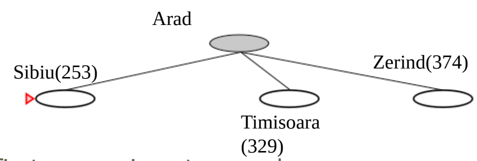



Assume that we want to use greedy search to solve the problem of travelling from Arad to Bucharest.

The initial state=Arad



## Greedy search example



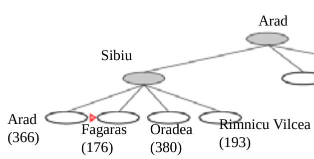

The first expansion step produces:

- Sibiu, Timisoara and Zerind

Greedy best-first will select Sibiu.



## Greedy search example



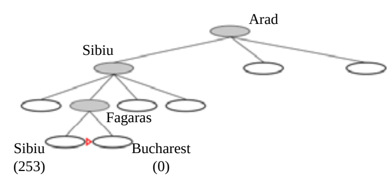

If Sibiu is expanded we get:

- Arad, Fagaras, Oradea and Rimnicu Vilcea

Greedy best-first search will select: Fagaras



## Greedy search example




If Fagaras is expanded we get:

- Sibiu and Bucharest

Goal reached !!

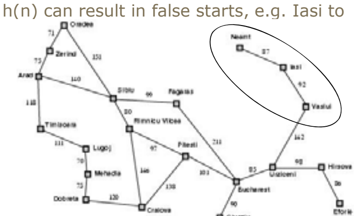
- Yet not optimal (see Arad, Sibiu, Rimnicu Vilcea, Pitesti)



## Greedy search, evaluation

Completeness: NO (cfr. DF-search)

- Check on repeated states
- Minimizing h(n) can result in false starts, e.g. Iasi to Fagaras.





## Greedy search, evaluation

Completeness: NO (cfr. DF-search)

Time complexity?

- Cfr. Worst-case DF-search $O(b^m)$

(with m is maximum depth of search space)

- Good heuristic can give dramatic improvement.

## Greedy search, evaluation

Completeness: NO (cfr. DF-search)

Time complexity: $O(b^m)$

Space complexity: $O(b^m)$

- Keeps all nodes in memory



## Greedy search, evaluation

Completeness: NO (cfr. DF-search)

Time complexity: $O(b^m)$

Space complexity: $O(b^m)$

Optimality? NO

- Same as DF-search



## A* search

Best-known form of best-first search.

Idea: avoid expanding paths that are already expensive.

Evaluation function *f(n)=g(n) + h(n)*

- *g(n)* the cost (so far) to reach the node.
- *h(n)* estimated cost to get from the node to the goal.
- *f(n)* estimated total cost of path through *n* to goal.



## A* search

A* search uses an admissible heuristic

- A heuristic is admissible if it *never overestimates* the cost to reach the goal
- Are optimistic

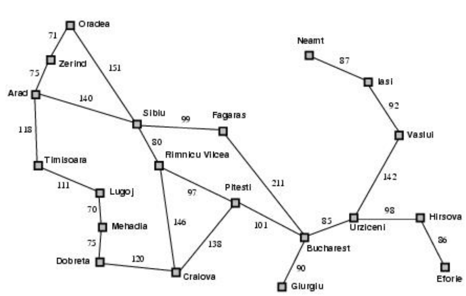
Formally:

1. *h(n) <= h*(n)* where *h*(n)* is the true cost from *n*
2. *h(n) >= 0* so *h(G)=0* for any goal *G*.

e.g. $h_{SLD}(n)$ never overestimates the actual road distance



## Romania example





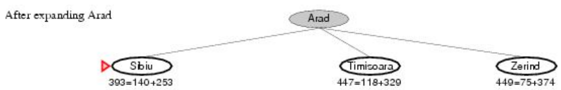

## A* search example

(a) The initial state


Find Bucharest starting at Arad

- f(Arad) = c(??,Arad)+h(Arad)=0+366=366

## A* search example



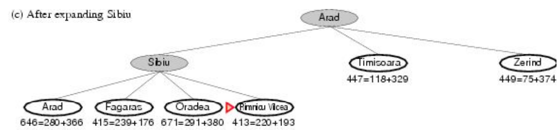

Expand Arrad and determine *f(n)* for each node

- f(Sibiu)=c(Arad,Sibiu)+h(Sibiu)=140+253=393
- f(Timisoara)=c(Arad,Timisoara)+h(Timisoara)=118+329=447
- f(Zerind)=c(Arad,Zerind)+h(Zerind)=75+374=449

Best choice is Sibiu



## A* search example



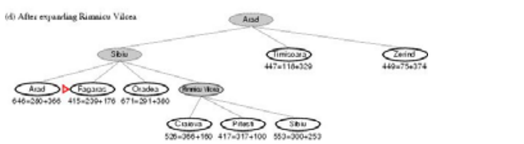

Expand Sibiu and determine *f(n)* for each node

- f(Arad)=c(Sibiu,Arad)+h(Arad)=280+366=646
- f(Fagaras)=c(Sibiu,Fagaras)+h(Fagaras)=239+179=415
- f(Oradea)=c(Sibiu,Oradea)+h(Oradea)=291+380=671
- f(Rimnicu Vilcea)=c(Sibiu,Rimnicu Vilcea)+
  h(Rimnicu Vilcea)=220+192=413

Best choice is Rimnicu Vilcea



## A* search example



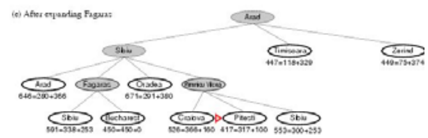
Expand Rimnicu Vilcea and determine *f(n)* for each node

- f(Craiova)=c(Rimnicu Vilcea, Craiova)+h(Craiova)=360+160=526
- f(Pitesti)=c(Rimnicu Vilcea, Pitesti)+h(Pitesti)=317+100=417
- f(Sibiu)=c(Rimnicu Vilcea,Sibiu)+h(Sibiu)=300+253=553

Best choice is Fagaras



## A* search example



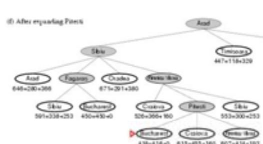
Expand Fagaras and determine *f(n)* for each node

- f(Sibiu)=c(Fagaras, Sibiu)+h(Sibiu)=338+253=591
- *f(Bucharest)=c(Fagaras,Bucharest)+h(Bucharest)=450+0=450*

Best choice is Pitesti !!!



## A* search example



Expand Pitesti and determine *f(n)* for each node

- *f(Bucharest)=c(Pitesti,Bucharest)+h(Bucharest)=418+0=418*

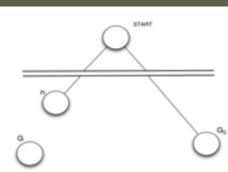
Best choice is Bucharest !!!

- Optimal solution (only if *h(n)* is admissable)

Note values along optimal path !!



## Optimality of A*(standard proof)



Suppose suboptimal goal $G_2$ in the queue.

Let $n$ be an unexpanded node on a shortest to optimal goal $G$.

$f(G_2) = g(G_2)$ since $h(G_2)=0$

$> g(G)$ since $G_2$ is suboptimal

$>= f(n)$ since $h$ is admissible

Since $f(G_2) > f(n)$, A* will never select $G_2$ for expansion

## BUT … graph search

Discards new paths to repeated state.

- Previous proof breaks down

Solution:

- Add extra bookkeeping i.e. remove more expsive of two paths.
- Ensure that optimal path to any repeated state is always first followed.
  - Extra requirement on *h(n)*: consistency (monotonicity)



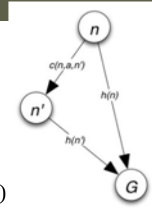
## Consistency

A heuristic is consistent if

$$h(n) \le c(n,a,n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\ge g(n) + h(n) \\ &\ge f(n) \end{aligned}$$




i.e. f(n) is nondecreasing along any path.



## Optimality of A*(more usefull)

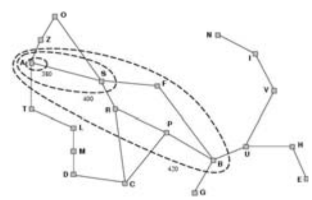
A* expands nodes in order of increasing *f* value

Contours can be drawn in state space

- Uniform-cost search adds circles.

- F-contours are gradually Added:
  1) nodes with $f(n)<C^*$
  2) Some nodes on the goal Contour ($f(n)=C^*$).

  Contour I has all Nodes with $f=f_i$, where $f_i < f_i+1$.





## A* search, evaluation

Completeness: YES

- Since bands of increasing *f* are added
- Unless there are infinitly many nodes with $f<f(G)$



## A* search, evaluation

Completeness: YES

Time complexity:

- Number of nodes expanded is still exponential in the length of the solution.



## A* search, evaluation

Completeness: YES

Time complexity: (exponential with path length)

Space complexity:

- It keeps all generated nodes in memory
- Hence space is the major problem not time

# A* search, evaluation

- Completeness: YES
- Time complexity: (exponential with path length)
- Space complexity:(all nodes are stored)
- Optimality: YES
  - Cannot expand $f_{i+1}$ until $f_i$ is finished.
  - A* expands all nodes with $f(n)< C^*$
  - A* expands some nodes with $f(n)=C^*$
  - A* expands no nodes with $f(n)>C^*$

Also *optimally efficient* (not including ties)



# Memory-bounded heuristic search

Some solutions to A* space problems (maintain completeness and optimality)

- Iterative-deepening A* (IDA*)
  - Here cutoff information is the *f*-cost (*g+h*) instead of depth
- Recursive best-first search(RBFS)
  - Recursive algorithm that attempts to mimic standard best-first search with linear space.
- (simple) Memory-bounded A* ((S)MA*)
  - Drop the worst-leaf node when memory is full



# Recursive best-first search

```
function RECURSIVE-BEST-FIRST-SEARCH(problem) return a solution or failure
    return RFBS(problem,MAKE-NODE(INITIAL-STATE[problem]),∞)

function RFBS( problem, node, f_limit) return a solution or failure and a new f-
    cost limit
    if GOAL-TEST[problem](STATE[node]) then return node
    successors ← EXPAND(node, problem)
    if successors is empty then return failure, ∞
    for each s in successors do
        f [s] ← max(g(s) + h(s), f [node])
    repeat
        best ← the lowest f-value node in successors
        if f [best] > f_limit then return failure, f [best]
        alternative ← the second lowest f-value among successors
        result, f [best] ← RBFS(problem, best, min(f_limit, alternative))
        if result ≠ failure then return result
```



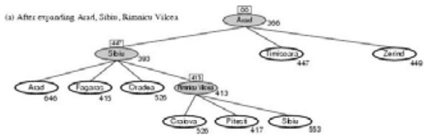
# Recursive best-first search

Keeps track of the f-value of the best-alternative path available.

- If current f-values exceeds this alternative f-value than backtrack to alternative path.
- Upon backtracking change f-value to best f-value of its children.
- Re-expansion of this result is thus still possible.



# Recursive best-first search, ex.




Path until Rumnicu Vilcea is already expanded

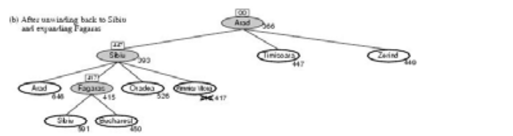
Above node; *f*-limit for every recursive call is shown on top.

Below node: *f(n)*

The path is followed until Pitesti which has a *f*-value worse than the *f-limit*.



# Recursive best-first search, ex.



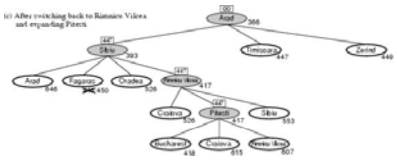

Unwind recursion and store best *f*-value for current best leaf Pitesti

*result*, *f* [*best*] ← RBFS(*problem, best,* min(*f_limit, alternative*))

*best* is now Fagaras. Call RBFS for new *best*

- *best* value is now 450

## Recursive best-first search, ex.



Unwind recursion and store best *f*-value for current best leaf Fagaras

*result*, *f* [*best*] ← RBFS(*problem*, *best*, min(*f_limit*, *alternative*))

*best* is now Rimnicu Viclea (again). Call RBFS for new *best*
- Subtree is again expanded.
- Best *alternative* subtree is now through Timisoara.

Solution is found since because 447 > 417.



## RBFS evaluation

RBFS is a bit more efficient than IDA*
- Still excessive node generation (mind changes)

Like A*, optimal if *h(n)* is admissible

Space complexity is *O(bd)*.
- IDA* retains only one single number (the current f-cost limit)

Time complexity difficult to characterize
- Depends on accuracy if h(n) and how often best path changes.

IDA* en RBFS suffer from ***too little*** memory.



## (simplified) memory-bounded A*

Use all available memory.
- I.e. expand best leafs until available memory is full
- When full, SMA* drops worst leaf node (highest *f*-value)
- Like RFBS backup forgotten node to its parent

What if all leafs have the same *f*-value?
- Same node could be selected for expansion and deletion.
- SMA* solves this by expanding *newest* best leaf and deleting *oldest* worst leaf.

SMA* is complete if solution is reachable, optimal if optimal solution is reachable.



## Learning to search better

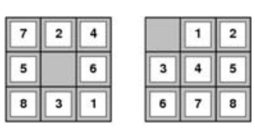
All previous algorithms use *fixed strategies*.

Agents can learn to improve their search by exploiting the *meta-level state space*.
- Each meta-level state is a internal (computational) state of a program that is searching in *the object-level state space*.
- In A* such a state consists of the current search tree

A meta-level learning algorithm from experiences at the meta-level.



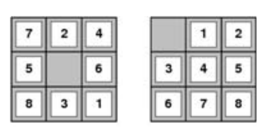
## Heuristic functions




E.g for the 8-puzzle
- Avg. solution cost is about 22 steps (branching factor +/- 3)
- Exhaustive search to depth 22: 3.1 x $10^{10}$ states.
- A good heuristic function can reduce the search process.



## Heuristic functions




E.g for the 8-puzzle knows two commonly used heuristics

$h_1$ = the number of misplaced tiles
- $h_1(s)=8$

$h_2$ = the sum of the distances of the tiles from their goal positions (manhattan distance).
- $h_2(s)=3+1+2+2+2+3+3+2=18$

# Heuristic quality

## Effective branching factor b*

- Is the branching factor that a uniform tree of depth *d* would have in order to contain *N+1* nodes.

$$N+1=1+b^*+(b^*)^2+\ldots+(b^*)^d$$

- Measure is fairly constant for sufficiently hard problems.
  - Can thus provide a good guide to the heuristic's overall usefulness.
  - A good value of b* is 1.



# Heuristic quality and dominance

1200 random problems with solution lengths from 2 to 24.

| | Search Cost | | | Effective Branching Factor | | |
|---|---|---|---|---|---|---|
| d | IDS | $A^*(h_1)$ | $A^*(h_2)$ | IDS | $A^*(h_1)$ | $A^*(h_2)$ |
| 2 | 10 | 6 | 6 | 2.45 | 1.79 | 1.79 |
| 4 | 112 | 13 | 12 | 2.87 | 1.48 | 1.45 |
| 6 | 680 | 20 | 18 | 2.73 | 1.34 | 1.30 |
| 8 | 6384 | 39 | 25 | 2.80 | 1.33 | 1.24 |
| 10 | 47127 | 93 | 39 | 2.79 | 1.38 | 1.22 |
| 12 | 3644035 | 227 | 73 | 2.78 | 1.42 | 1.24 |
| 14 | – | 539 | 113 | – | 1.44 | 1.23 |
| 16 | – | 1301 | 211 | – | 1.45 | 1.25 |
| 18 | – | 3056 | 363 | – | 1.46 | 1.26 |
| 20 | – | 7276 | 676 | – | 1.47 | 1.27 |
| 22 | – | 18094 | 1219 | – | 1.48 | 1.28 |
| 24 | – | 39135 | 1641 | – | 1.48 | 1.26 |

If $h_2(n) >= h_1(n)$ for all *n* (both admissible) then $h_2$ *dominates* $h_1$ and is better for search



# Inventing admissible heuristics

Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem:

- Relaxed 8-puzzle for $h_1$ : a tile can move anywhere
  - As a result, $h_1(n)$ gives the shortest solution
- Relaxed 8-puzzle for $h_2$ : a tile can move to any adjacent square.
  - As a result, $h_2(n)$ gives the shortest solution.

The optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem.

`ABSolver` found a usefull heuristic for the rubic cube.



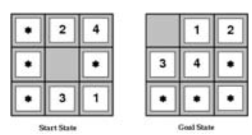
# Inventing admissible heuristics

Admissible heuristics can also be derived from the solution cost of a subproblem of a given problem.

This cost is a lower bound on the cost of the real problem.

Pattern databases store the exact solution to for every possible subproblem instance.

- The complete heuristic is constructed using the patterns in the DB






# Inventing admissible heuristics

Another way to find an admissible heuristic is through learning from experience:

- Experience = solving lots of 8-puzzles
- An inductive learning algorithm can be used to predict costs for other states that arise during search.



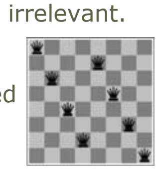
# Local search and optimization

Previously: systematic exploration of search space.

- Path to goal is solution to problem

YET, for some problems path is irrelevant.

- E.g 8-queens



Different algorithms can be used

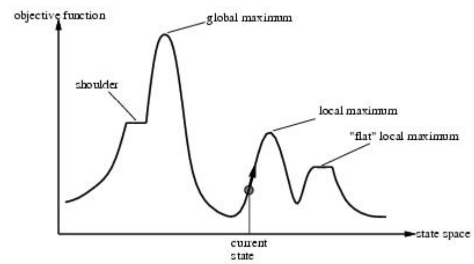
- Local search

## Local search and optimization

- Local search= use single current state and move to neighboring states.
- Advantages:
  - Use very little memory
  - Find often reasonable solutions in large or infinite state spaces.
- Are also useful for pure optimization problems.
  - Find best state according to some *objective function*.
  - e.g. survival of the fittest as a metaphor for optimization.



## Local search and optimization






## Hill-climbing search

- "is a loop that continuously moves in the direction of increasing value"
  - It terminates when a peak is reached.
- Hill climbing does not look ahead of the immediate neighbors of the current state.
- Hill-climbing chooses randomly among the set of best successors, if there is more than one.
- Hill-climbing a.k.a. *greedy local search*



## Hill-climbing search

**function** HILL-CLIMBING( *problem*) **return** a state that is a local maximum
 **input:** *problem*, a problem
 **local variables:** *current*, **a node.**
  *neighbor*, **a node.**

 *current* ← MAKE-NODE(INITIAL-STATE[*problem*])
 **loop do**
  *neighbor* ← a highest valued successor of *current*
  **if** VALUE [*neighbor*] ≤ VALUE[*current*] **then return** STATE[*current*]
  *current* ← *neighbor*

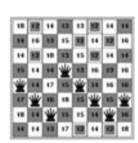


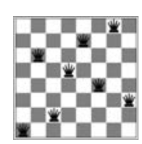
## Hill-climbing example

- 8-queens problem (complete-state formulation).
- Successor function: move a single queen to another square in the same column.
- Heuristic function *h(n)*: the number of pairs of queens that are attacking each other (directly or indirectly).



## Hill-climbing example

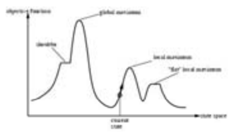
a)


b)


a) shows a state of h=17 and the h-value for each possible successor.

b) A local minimum in the 8-queens state space (h=1).

## Drawbacks



Ridge = sequence of local maxima difficult for greedy algorithms to navigate

Plateaux = an area of the state space where the evaluation function is flat.

Gets stuck 86% of the time.



## Hill-climbing variations

Stochastic hill-climbing
- Random selection among the uphill moves.
- The selection probability can vary with the steepness of the uphill move.

First-choice hill-climbing
- cfr. stochastic hill climbing by generating successors randomly until a better one is found.

Random-restart hill-climbing
- Tries to avoid getting stuck in local maxima.



## Simulated annealing

Escape local maxima by allowing "bad" moves.
- Idea: but gradually decrease their size and frequency.

Origin; metallurgical annealing

Bouncing ball analogy:
- Shaking hard (= high temperature).
- Shaking less (= lower the temperature).

If T decreases slowly enough, best state is reached.

Applied for VLSI layout, airline scheduling, etc.



## Simulated annealing

**function** SIMULATED-ANNEALING( *problem, schedule*) **return** a solution state
  **input:** *problem*, a problem
    *schedule*, a mapping from time to temperature
  **local variables:** *current*, a node.
    *next*, a node.
    *T*, a "temperature" controlling the probability of downward steps

  *current* ← MAKE-NODE(INITIAL-STATE[*problem*])
  **for t ← 1 to ∞ do**
    *T* ← *schedule*[*t*]
    **if** *T = 0* **then return** *current*
    *next* ← a randomly selected successor of *current*
    *ΔE* ← VALUE[*next*] - VALUE[*current*]
    **if** *ΔE* > 0 **then** *current* ← *next*
    **else** *current* ← *next* only with probability $e^{\Delta E/T}$



## Local beam search

Keep track of *k* states instead of one
- Initially: *k* random states
- Next: determine all successors of *k* states
- If any of successors is goal → finished
- Else select *k* best from successors and repeat.

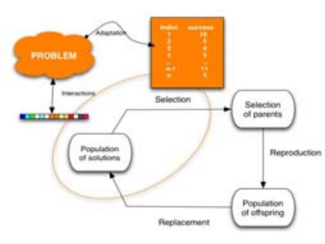
Major difference with random-restart search
- Information is shared among *k* search threads.

Can suffer from lack of diversity.
- Stochastic variant: choose k successors at proportionally to state success.



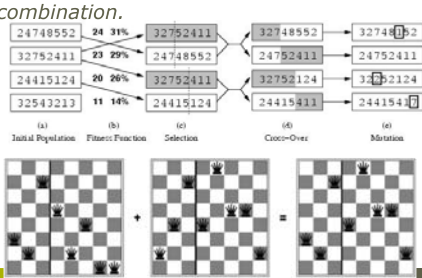
## Genetic algorithms

Variant of local beam search with *sexual recombination.*



## Genetic algorithms

Variant of local beam search with *sexual recombination.*






## Genetic algorithm

**function** GENETIC_ALGORITHM( *population*, FITNESS-FN) **return** an individual
**input:** *population*, a set of individuals
FITNESS-FN, a function which determines the quality of the individual
**repeat**
*new_population* ← empty set
**loop for** i **from** 1 **to** SIZE(*population*) **do**
*x* ← RANDOM_SELECTION(*population*, FITNESS_FN)
*y* ← RANDOM_SELECTION(*population*, FITNESS_FN)
*child* ← REPRODUCE(*x*,*y*)
**if** (small random probability) **then** *child* ← MUTATE(*child* )
add *child* to *new_population*
*population* ← *new_population*
**until** some individual is fit enough or enough time has elapsed
**return** the best individual



## Exploration problems

Until now all algorithms were offline.
- Offline= solution is determined before executing it.
- Online = interleaving computation and action

Online search is necessary for dynamic and semi-dynamic environments
- It is impossible to take into account all possible contingencies.

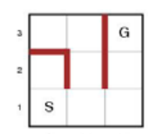
Used for *exploration problems*:
- Unknown states and actions.
- e.g. any robot in a new environment, a newborn baby,…



## Online search problems

Agent knowledge:
- ACTION(s): list of allowed actions in state s
- C(s,a,s'): step-cost function (! After s' is determined)
- GOAL-TEST(s)

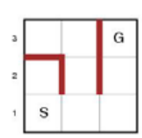
An agent can recognize previous states.
Actions are deterministic.
Access to admissible heuristic *h(s)*
e.g. manhattan distance





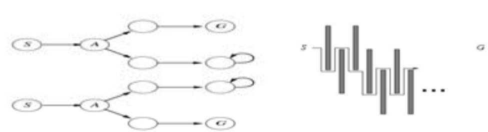

## Online search problems

Objective: reach goal with minimal cost
- Cost = total cost of travelled path
- Competitive ratio=comparison of cost with cost of the solution path if search space is known.
- Can be infinite in case of the agent accidentally reaches dead ends






## The adversary argument




Assume an adversary who can construct the state space while the agent explores it
- Visited states S and A. What next?
  - Fails in one of the state spaces

No algorithm can avoid dead ends in all state spaces.

## Online search agents

The agent maintains a map of the environment.

- Updated based on percept input.
- This map is used to decide next action.

Note difference with e.g. A*

An online version can only expand the node it is physically in (local order)



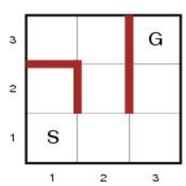
## Online DF-search

**function** ONLINE_DFS-AGENT(*s′*) **return** an action
  **input:** *s′*, a percept identifying current state
  **static**: *result*, a table indexed by action and state, initially empty
    *unexplored*, a table that lists for each visited state, the action not yet tried
    *unbacktracked*, a table that lists for each visited state, the backtrack not yet tried
    *s,a*, the previous state and action, initially null

  **if** GOAL-TEST(*s′*) **then return** stop
  **if** *s′* is a new state **then** *unexplored*[*s′*] ← ACTIONS(*s′*)
  **if** *s* is not null **then do**
    *result*[*a,s*] ← *s′*
    add *s* to the front of *unbackedtracked*[*s′*]
  **if** *unexplored*[*s′*] is empty **then**
    **if** *unbacktracked*[*s′*] is empty **then return** stop
    **else** *a* ← an action *b* such that *result*[*b*, *s′*]=POP(*unbacktracked*[*s′*])
  **else** *a* ← POP(*unexplored*[*s′*])
  s ← s′
  **return** *a*



## Online DF-search, example




Assume maze problem on 3x3 grid.

s′ = (1,1) is initial state

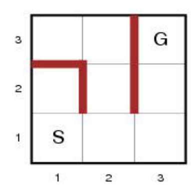
Result, unexplored (UX), unbacktracked (UB), …
  are empty

S,a are also empty



## Online DF-search, example

S'=(1,1)




GOAL-TEST((,1,1))?
- S not = G thus false

(1,1) a new state?
- True
- ACTION((1,1)) -> UX[(1,1)]
  - {RIGHT,UP}

s is null?
- True (initially)

UX[(1,1)] empty?
- False

POP(UX[(1,1)])->a
- A=UP

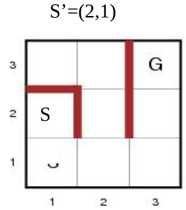
s = (1,1)

Return a



## Online DF-search, example

S'=(2,1)




GOAL-TEST((2,1))?
- S not = G thus false

(2,1) a new state?
- True
- ACTION((2,1)) -> UX[(2,1)]
  - {DOWN}

s is null?
- false (s=(1,1))
- result[UP,(1,1)] <- (2,1)
- UB[(2,1)]={(1,1)}

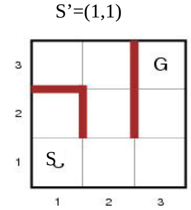
UX[(2,1)] empty?
- False

A=DOWN, s=(2,1) return A



## Online DF-search, example

S'=(1,1)




GOAL-TEST((1,1))?
- S not = G thus false

(1,1) a new state?
- false

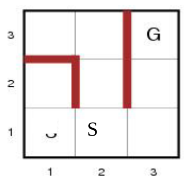
s is null?
- false (s=(2,1))
- result[DOWN,(2,1)] <- (1,1)
- UB[(1,1)]={(2,1)}

UX[(1,1)] empty?
- False

A=RIGHT, s=(1,1) return A

## Online DF-search, example

S’=(1,2)




GOAL-TEST((1,2))?
- S not = G thus false

(1,2) a new state?
- True, UX[(1,2)]={RIGHT,UP,LEFT}

s is null?
- false (s=(1,1))
- result[RIGHT,(1,1)] <- (1,2)
- UB[(1,2)]={(1,1)}

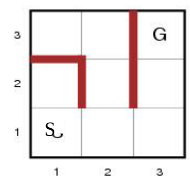
UX[(1,2)] empty?
- False

A=LEFT, s=(1,2) return A



## Online DF-search, example

S’=(1,1)




GOAL-TEST((1,1))?
- S not = G thus false

(1,1) a new state?
- false

s is null?
- false (s=(1,2))
- result[LEFT,(1,2)] <- (1,1)
- UB[(1,1)]={(1,2),(2,1)}

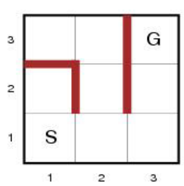
UX[(1,1)] empty?
- True
- UB[(1,1)] empty? False

A= b for b in result[b,(1,1)]=(1,2)
- B=RIGHT

A=RIGHT, s=(1,1) …



## Online DF-search




Worst case each node is visited twice.

An agent can go on a long walk even when it is close to the solution.

An online iterative deepening approach solves this problem.

Online DF-search works only when actions are reversible.

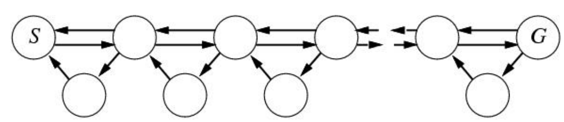


## Online local search

Hill-climbing is already online
- One state is stored.

Bad performancd due to local maxima
- Random restarts impossible.

Solution: Random walk introduces exploration (can produce exponentially many steps)

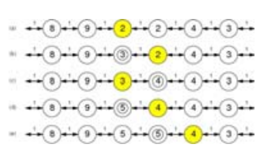





## Online local search

Solution 2: Add memory to hill climber
- Store current best estimate *H(s)* of cost to reach goal
- *H(s)* is initially the heuristic estimate *h(s)*
- Afterward updated with experience (see below)

Learning real-time A* (LRTA*)





## Learning real-time A*

```
function LRTA*-COST(s,a,s',H) return an cost estimate
    if s' is undefined the return h(s)
    else return c(s,a,s') + H[s']
function LRTA*-AGENT(s') return an action
    input: s', a percept identifying current state
    static: result, a table indexed by action and state, initially empty
            H, a table of cost estimates indexed by state, initially empty
            s,a, the previous state and action, initially null

    if GOAL-TEST(s')  then return stop
    if s' is a new state (not in H) then H[s'] ← h(s')
    unless  s is null
            result[a,s] ←  s'
            H[s] ←     MIN          LRTA*-COST(s,b,result[b,s],H)
                   b ∈ ACTIONS(s)
     a ← an action b in ACTIONS(s') that minimizes LRTA*-COST(s',b,result[b,s'],H)
    s ←  s'
    return a
```